MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

Illustration by Mike Avitabile

Is there any difference between a modal test with a shaker excitation or impact excitation?
Well ... that's a good question.
The answer is yes and no.

This is another question that gets asked often. There's a lot of different aspects relating to this. Let's start with some basics to understand why it is so difficult to answer this question as either yes or no. A few simple equations are needed to help explain this.

First, we have to remember that any system can be described by it's equation of motion. Basically, the equation is simply the force balance of mass times acceleration plus damping times velocity plus stiffness times displacement which is equal to the applied force. For a number of reasons, it is easier to work with this equation in the Laplace domain. By taking the Laplace transform of the equation of motion, we can write

$$\Big[[M]s^2+[C]s+[K]\Big]\{X(s)\}=\{F(s)\}\Rightarrow[B(s)]\{X(s)\}=\{F(s)\}$$

We use matrices to help organize all of the equations. Remember that [M], [C], [K] are the mass, damping and stiffness matrices respectively. It is very important to note that these matrices are symmetric. Therefore, the system matrix, [B(s)], is also symmetric. The system transfer function is the inverse of the system matrix given by

$$[B(s)]^{-1}=[H(s)]=\frac{\mathrm{Adj}[B(s)]}{\det[B(s)]}=\frac{[A(s)]}{\det[B(s)]}$$

And, of course, you remember that the frequency response function that we measure during a modal test is nothing more than the system transfer function evaluated along the frequency axis. Most of the time, we write the frequency response function in partial fraction form, for convenience, as

$$[H(s)]_{s=j\omega}=[H(j\omega)]=\sum_{k=1}^{m}\frac{[A_k]}{(j\omega-p_k)}+\frac{[A_k^*]}{(j\omega-p_k^*)}$$

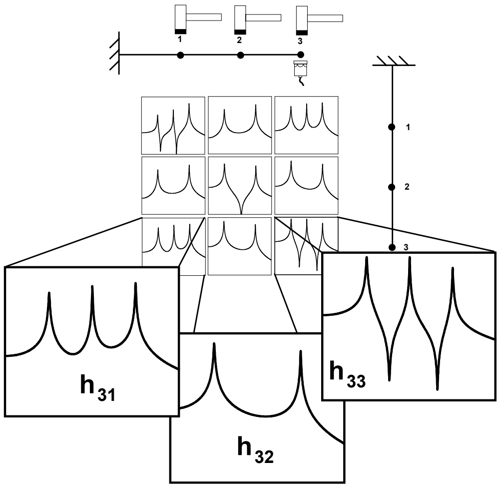
and an individual term can be written as

$$h_{ij}(j\omega)=\sum_{k=1}^{m}\frac{a_{ijk}}{(j\omega-p_k)}+\frac{a_{ijk}^*}{(j\omega-p_k^*)}$$

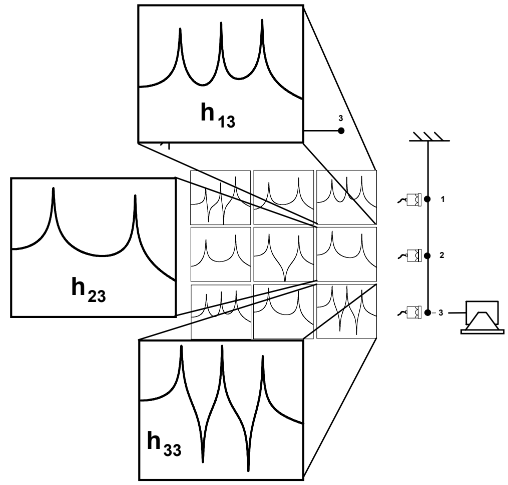
So why did I bother writing out all these equations? That's because there are some very important things to note in these equations relative to your question. Remember that [B(s)] and [H(s)] are symmetric since [M], [C], and [K] are symmetric. That means that [H(jω)] is also symmetric. This implies that $h_{ij}=h_{ji}$ which is called reciprocity. This means that you can measure the FRF by impacting point 'i' and measuring the response at point 'j' and get exactly the same FRF as impacting point 'j' and measuring the response at point 'i'. This is what is meant by reciprocity.

Now, let's consider an impact test situation for a simple beam with three measurement locations. There are a total of nine possible input-output FRFs that could be measured. But for this case, let's put our accelerometer at point 3 and make FRF measurements by impacting the beam at point 1, 2, and 3. We call point 3 the reference location since it is the same response point for each of the measurements that I make. Since the hammer is roving from one point to another point, the FRFs that are measured come from one row of the FRF matrix, the last row of the matrix.

Impact Test Measurements



Shaker Test Measurements



Before we talk about anything else, let's discuss the same set of measurements from a shaker test. Let's place our shaker at point 3 and make FRF measurements by roving the accelerometer to point 1, 2, and 3 on the beam; note that point 3 is still the reference location since the force is applied to the same point for each measurement. Now that the force is stationary, the FRFs that are measured come from one column of the FRF matrix, the last column of the matrix.

If I look at the measurements taken, I'll notice that h_{13} from the shaker test is *exactly the same* as h_{31} from the impact test. Also notice that h_{23} from the shaker test is *exactly the same* as h_{32} from the impact test. Well, this is what reciprocity is all about. So, from a theoretical standpoint, it doesn't matter whether I collect data from a shaker test or an impact test. The data is exactly the same - from a theoretical standpoint. If fact, there is no reason why the impact test can't be performed by impacting the same point on the structure and roving the accelerometer around to all the different measurement locations. I could draw the same analogy for the shaker test also. We could keep the response accelerometer at the same location and move the shaker from point to point (but I don't know anyone who wants to run a test that way!) The point is that from a theoretical standpoint, it doesn't matter how the data is collected as long as the input-output characteristics are obtained.

So the answer is that there is no difference between a shaker test and an impact test. That is, from a theoretical standpoint! If I can apply pure forces to a structure without any interaction between the applied force and the structure and I can measure response with a massless transducer that has no effect on the structure - then this is true. But what if this is not the case.

Now let's think about performing the test from a practical standpoint. The point is that shakers and response transducers generally do have an effect on the structure during the modal test. The main item to remember is that the structure under test is not just the structure that you would like to obtain modal data. It is the structure plus everything involved in the acquisition of the data - the structure suspension, the mass of the mounted transducers, the potential stiffening effects of the shaker/stinger arrangement, etc. So while theory tells me that there shouldn't be any difference between the impact test results and the shaker test results, often there will be differences due to the practical aspects of collecting data.

The most obvious difference will occur from the roving of accelerometers during a shaker test. The weight of the accelerometer may be extremely small relative to the total weight of the whole structure, but it's weight may be quite large relative to the effective weight of different parts of the structure. This is accentuated in multi-channel systems where many accelerometers are moved around the structure in order to acquire all the measurements. This can be a problem especially on light weight structures. One way to correct this problem is to mount all of the accelerometers on the structure even though only a few are measured at a time. Another way is to add dummy accelerometer masses at locations not being measured; this will eliminate the roving mass effect.

Another difference that can result is due to the shaker/stinger effects. Basically, the modes of the structure may be affected by the mass and stiffness effects of the shaker attachment. While we try to minimize these effects, they may exist. The purpose of the stinger is to divorce the effects of the shaker from the structure. However, on many structures, the effects of the shaker attachment may be significant. Since an in impact test does not suffer from these problems, different results may be obtained.

So while theory says that there is no difference between a shaker test and an impact test, there are some very basic practical aspects that may cause some differences. I hope this clears up this question.