MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

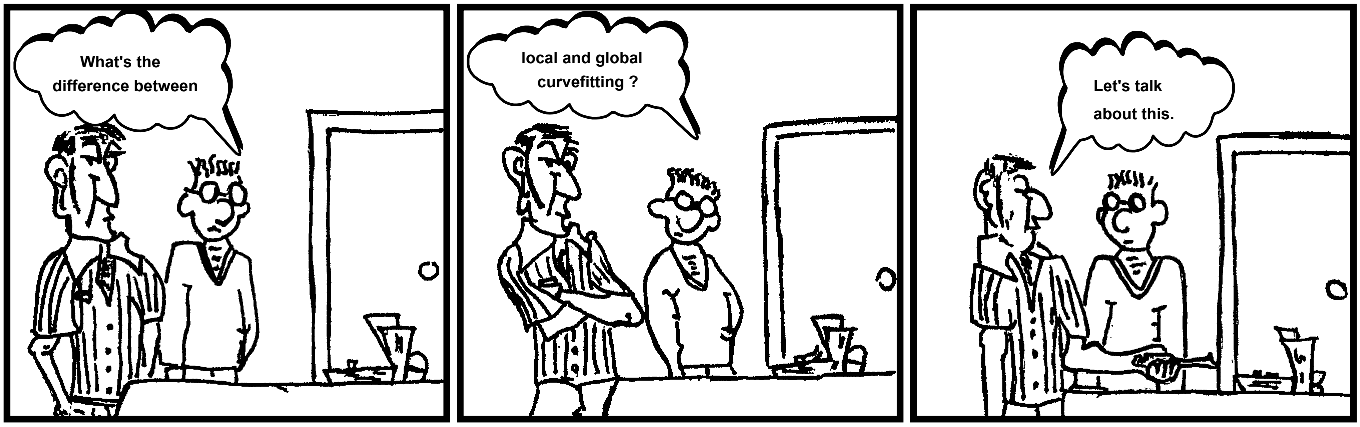

Illustration by Mike Avitabile

What's the difference between local and global curvefitting ??
Let's talk about this.

This is a good question. In order to explain this, a few quick equations are needed followed by a simple example that will illustrate the differences. Let's recall the frequency response function which is

$$h_{ij}(j\omega) = \sum_{k=1}^{m} \frac{a_{ijk}}{(j\omega - p_k)} + \frac{a_{ijk}^*}{(j\omega - p_k^*)}$$

There are numerous 'ij' (output-input) combinations; a matrix of possible FRFs is illustrated in Figure 1.

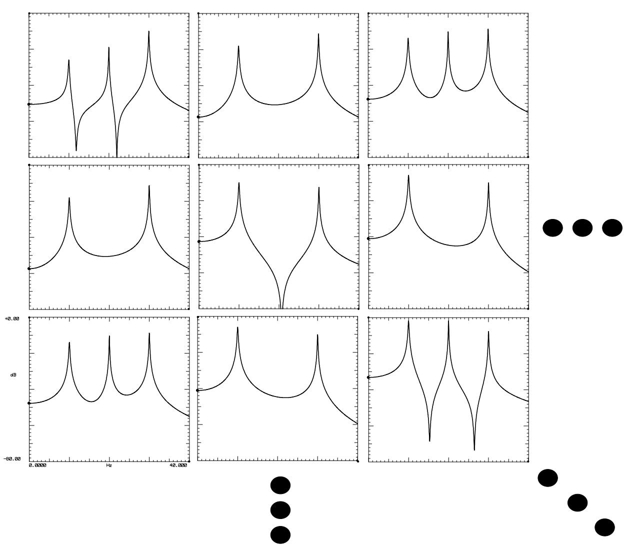

Figure 1 - FRF Matrix for a Multiple DOF System

Now each FRF is defined by poles and residues. The FRF is different from one measurement to the next because the residue is different. This is true since the mode shapes are related to the residues as

$$a_{ijk} = q_k u_{ik} u_{jk}$$

But it is very important to note that the denominator of the FRF is constant and does not change from one measurement to the next. Since the pole does not change from one measurement to the next, then it is said that the pole is a "global" property of the system. This means that while the residue changes from one measurement to the next, as expected, the pole does not change - at least theoretically! But in real measurements, this may not necessarily be the case. In actuality, the pole may shift from one measurement to the next. This can cause a problem.

To understand this, consider data to be fit with a straight line as shown in Figure 2. Now, if only two points are selected (blue) different from another set of points (red), there can be dramatic differences in the slope and y-intercept computed from the two sets of points. In other words, there are differences and inconsistencies in the slope and y-intercept depending on which data is used to extract parameters. When all the data is used together in a least squares fashion, then the "best" overall estimate of the slope and y-intercept results.

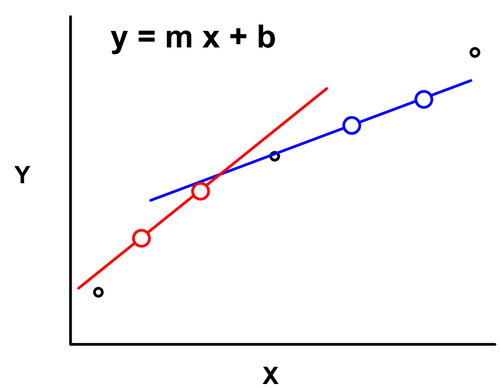

Figure 2 - Illustration on Parameter Variation

The same effect can be seen in the extraction of modal parameters from measured FRFs if each FRF is evaluated independently from every other FRF. Depending on which FRF is used, there may be differences in the estimated pole - but the theory indicates that this should not happen. However, this is exactly what happens when real measurements are used to extract modal parameters when each measurement is considered independently from each other. This is referred to as "local" curvefitting. In order to circumvent this problem, all of the measurements are used together, as one set, to find the best pole in a least squares fashion, to describe the best "global" representation of the pole. Once the pole is estimated, the residues are then estimated with the "global" estimate of the pole used in the modal parameter estimation equations. This is a two step process where the best "global" pole of the system is estimated first, followed by the estimation of residues with the estimated "global" pole of the system locked to a fixed value regardless of what each measurement may indicate. This is global curvefitting.

To illustrate the differences in local and global curvefitting, FRF measurements on a simple planar frame are used. Several FRFs are shown in Figure 3. There are 5 distinct modes in the band shown. Notice that the top two FRFs show all the peaks for each of the modes of the frame but that the lower two FRFs do not contain peaks at each one of the frequencies shown in the upper two plots. (This is due to the fact that some of the measurements are located at nodes of some of the modes.)

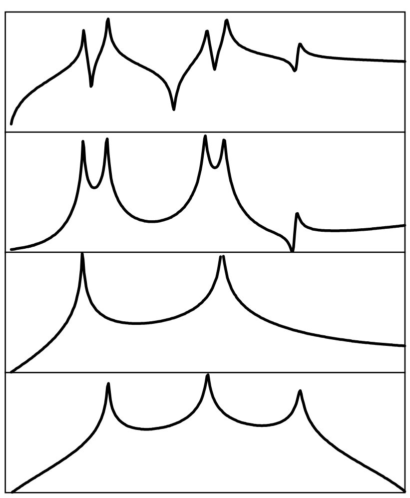

Figure 3 - Several FRFs from the Planar Frame

Now if there is no peak in a particular measurement, then how can pole values be estimated? This poses a serious problem and it is exactly these situations that the local curvefitting breaks down. If local curvefitting is performed on this type of data, then the estimated modal parameters may contain poorly extracted values from the individual FRF local curvefitting approach since the pole is estimated poorly. A local curvefitting technique was used to estimate modal parameters for the planar frame structure. The modes shapes are shown in Figure 4.

Notice that there are several locations in the mode shape where the data appears to be inconsistent from the expected mode shape. The modes shapes are distorted. It turns out that these points correspond to nodes of modes of the structure. (This is a well-known problem with local curvefitting.)

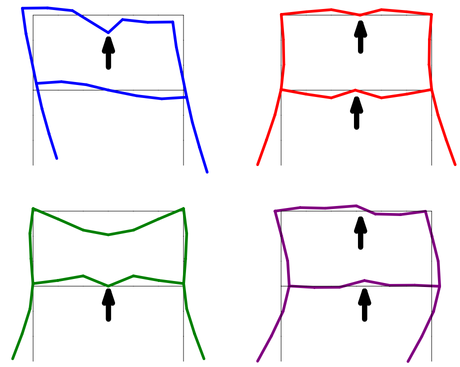

Figure 4 - Distorted Mode Shapes from Local Curvefitting

The same set of FRFs was used for global curvefitting. First, the best global pole of the system was estimated and then the residues were estimated in a second pass with the global pole used for all the FRF measurements when estimating residues. The mode shapes are shown in Figure 5. Notice that these mode shapes are the expected shapes of the planar frame structure.

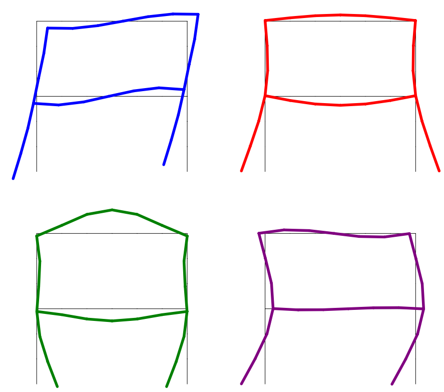

Figure 5 - Correct Mode Shapes from Global Curvefitting

Now from this example, it is clear that global curvefitting produces superior results. However, when collecting data, care must be exercised to assure that the data satisfies the requirement of global curvefitting - the modes must be global in all of the measurements collected! If the data is inconsistent, then errors may result in the estimation process. Care must be exercised to collect FRF data that satisfies the global nature necessary for the global data reduction process.

I hope this clears up your question. If you have any more questions on modal analysis, just ask me.