**MODAL SPACE - IN OUR OWN LITTLE WORLD** *by Pete Avitabile*

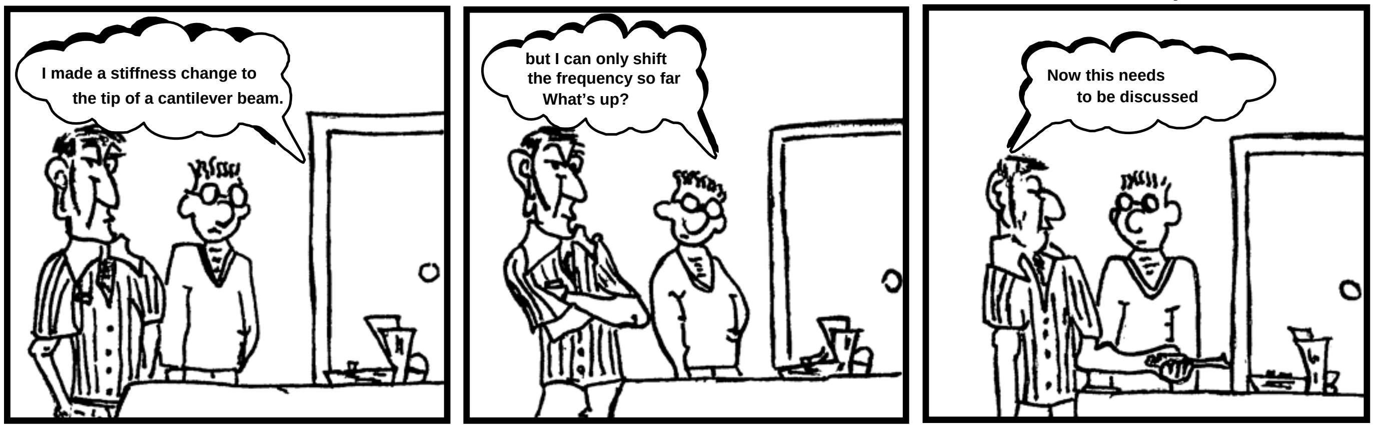


*Illustration by Mike Avitabile*

I made a stiffness change to the tip of a cantilever beam but I can only shift the frequency so far. What's up? Now this needs to be discussed.

OK. This is another one of those problems that I see many people get confused about. Let's start with a simple cantilever beam and explain some basic properties that are inherent in the system.

First, let's start with a simple finite element model to investigate the effects of stiffness at the tip of the cantilever beam. Figure 1 shows the cantilever beam along with the cantilever beam with a spring at the tip and the cantilever beam with the end pinned. A finite element model of the beam will be used to lend some insight into what happens when the spring at the tip of the beam is varied from low stiffness to high stiffness.

Table 1 shows the first three modes of the cantilever beam and then the change in frequency as the stiffness is increased along with the final pinned result if the spring was infinitely stiff. It is very important to notice that as the spring stiffness is increased, the final frequencies converge towards the final result where the cantilever is pinned at the tip.

So this implies that the no matter how much stiffness you add at the end of the cantilever beam, the frequency can only shift so far and then any additional increases in stiffness have very little effect at all – it is a point of diminishing returns.

Now let's further consider the simple cantilever beam and let's look at the tip response. The frequency response function is shown in Figure 2 with a drive point measurement at the tip of the beam where the stiffness is to be added to the beam.

So now let's look at the frequency response function and discuss the different parts of this function. At the natural frequencies, there is a peak in the function. Basically, this is a region in frequency where it takes very little force to cause large response. At the resonant frequency it appears that the structure has no apparent stiffness.

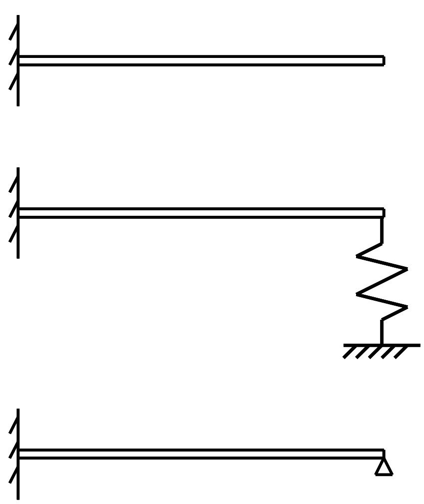

Figure 1 – Cantilever Beam, Cantilever Beam with Tip Spring, and Cantilever with Tip Pinned

Table 1: Cantilever Beam Frequencies with Various Tip Conditions (Free, Spring, Pinned)

| Condition | Mode 1 | Mode 2 | Mode 3 |
|---|---|---|---|
| Cantilever | 58. | 363. | 1017. |
| K=1E1 | 68. | 365. | 1018. |
| K=1E2 | 123. | 382. | 1024. |
| K=1E3 | 224. | 546. | 1092. |
| K=1E4 | 251. | 787. | 1544. |
| K=1E5 | 254. | 820. | 1705. |
| K=1E6 | 254. | 823. | 1718. |
| Pinned | 254. | 823. | 1720. |

Now at the antiresonances, this is a region in frequency where it takes excessive force and there is very little to essentially no response. At the antiresonant frequency, it appears that the structure is infinitely stiff. That is to say that at the antiresonant frequencies, there is no displacement and it appears that the cantilever is pinned at that point at that antiresonant frequency.

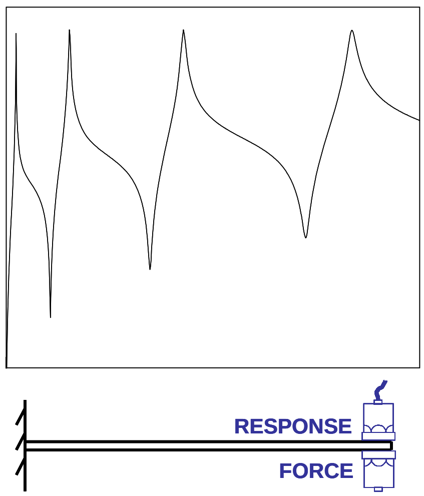


Figure 2 – Drive Point FRF Measurement for the Tip of the Cantilever Beam

Now if there would be a change in stiffness at the tip of the cantilever beam, then there would be a shift in the peaks of this function. If stiffness is added to the tip of the beam then the peaks will shift upward. This is shown in Figure 3.

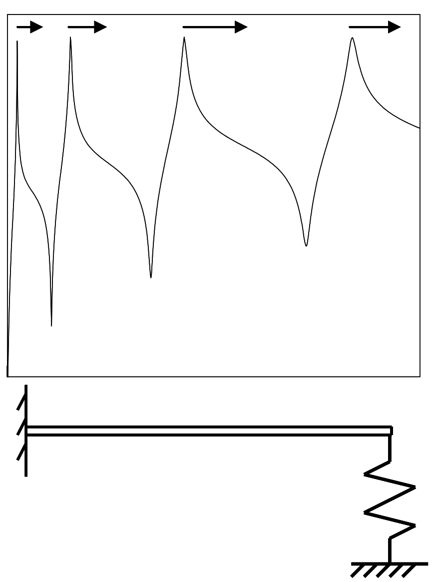

Figure 3 – Shift of Frequencies Due to Spring

But as the stiffness is increased, there will be some limit to the shift in the frequency of the system. Now if we realize that the antiresonance is actually the frequency at which the cantilever beam tip displacement is zero, then it is obvious that this is the frequency where the beam appears to be pinned at the tip. This is shown in Figure 4. From that schematic it is easy to realize that the peaks of the unconstrained cantilever beam can never shift past the antiresonances of the cantilever beam because this is essentially the cantilever constrained at the tip which is the pinned condition.

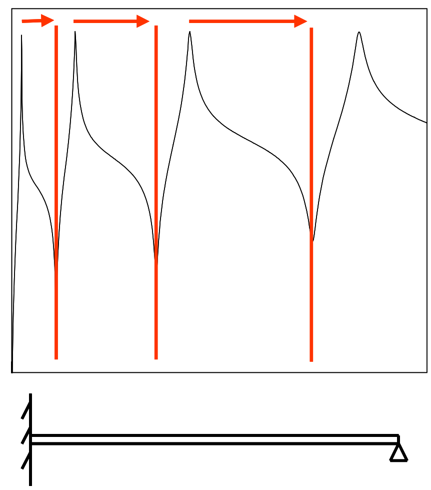

Figure 4 – Maximum Shift of Frequencies with Constraint

So from this discussion, it should be clear that the cantilever beam frequencies can only shift so far when a spring is considered at the tip of the beam. Further, we can actually identify how far those frequencies can shift by looking at the antiresonances at the tip of the unconstrained cantilever beam.

I hope that this discussion clears up the mystery as to why the frequencies can only shift so far before there is no further change in the frequencies. The best way to prove it to yourself is to make a simple finite element model and check out the results. If you have any more questions on modal analysis, just ask me.