MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

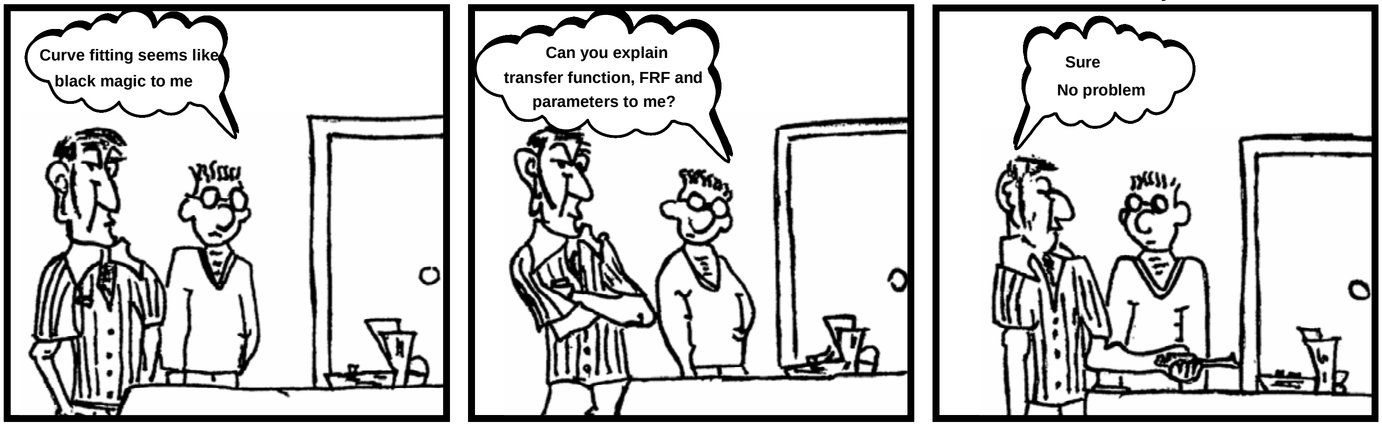


Illustration by Mike Avitabile

Curvefitting still seems like black magic to me. Can you explain transfer function, FRF and parameters to me?
Sure – no problem.

Well, curvefitting might look like black magic at first but I want to make a few simple analogies to help you understand that it is really fairly straight-forward and with the example I will show is very simple indeed.

The last time we discussed some related information regarding the system transfer function and the frequency response function (FRF). We wrote the system transfer function in partial fraction form for a single degree of freedom system as

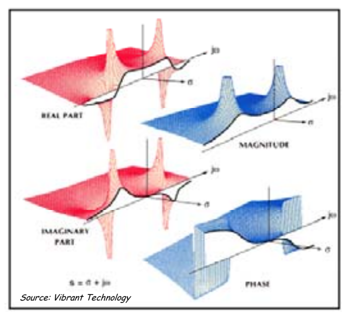
$$h(s)=\frac{a_1}{(s-p_1)}+\frac{a_1^*}{(s-p_1^*)}$$

and we also wrote the frequency response equation as

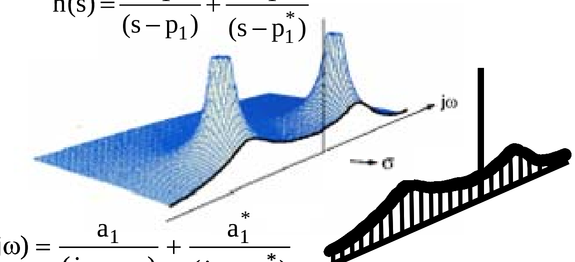
$$h(j\omega)=h(s)\Big|_{s=j\omega}=\frac{a_1}{(j\omega-p_1)}+\frac{a_1^*}{(j\omega-p_1^*)}$$

Now if we look at these two equations we notice in the first case the independent variable is "s" and in the second equation it is "ω" and that the "h" depends on these values. But I also notice that there are two constants, or parameters that are "a" the residue and "p" the pole. So these are the parameters that define "h" given some value "ω"; we call these modal parameters.

Now if we look at the system transfer function or a piece of the system transfer function called the frequency response function, we need to realize that the surface of the system transfer function as well as the curve of the frequency response function are defined by only two parameters for the single degree of freedom system, namely the pole "p" and residue "a". So looking at Figure 1, we need to realize that only two parameters define that surface and line – pretty amazing.



$$h(s)=\frac{a_1}{(s-p_1)}+\frac{a_1^*}{(s-p_1^*)}$$



$$h(j\omega)=\frac{a_1}{(j\omega-p_1)}+\frac{a_1^*}{(j\omega-p_1^*)}$$

Figure 1 – System Transfer Function and FRF

Now let's take a step back to something a little simpler and more commonly understood. Let's look at a very simple straight line fit of of some measured data. We are going to perform a least squares error minimization for the data presented in Figure 2. Now we know we can fit any line to the data but for this set of data it seems that a first order fit makes the most sense. Of course the model we are going to use is

$$y=mx+b$$

and there are two parameters that define the line, namely the slope and y intercept.

So for instance, in Figure 2 the resulting least squares fit of the data resulted in two parameters with a slope of 12.097 and a y-intercept of -0.019. Also realize that this data was obtained from a set of measured data that had some variance and that the least squares regression analysis identified the best parameters to represent this data with these two parameters of slope and y-intercept.

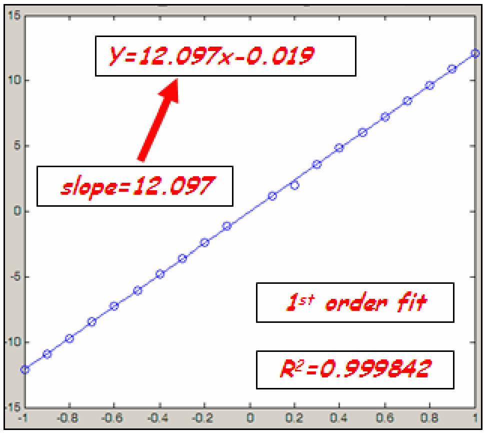


Figure 2 Example of Simple Straight Line Fit

So if we were to apply this same logic to a single degree of freedom frequency response function then I would fit a first order model of the form of a frequency response function as written above to the data presented in Figure 3. And if you looked at this schematic it would be very easy to see that there is a set of data and curvefit from which two parameters are obtatained, namely the pole and residue.

It is really the same as the straight line fit except that the data is complex and the line is a little more complicated. But in principle, it is the same methodology. We measure data at discrete data points as complex data and then fit a line of the frequency response function to the data to find the parameters that best describe the data in a least squares fashion.

Now of course the data in Figure 3 is for a single degree of freedom system. This same approach can be extended to a higher order function as shown in Figure 4. So in this way we can fit multiple modes (or essentially a higher order polynomial) to the data described by the discrete complex data measured from the frequency response function. And all the same arguments relating to the estimation of modal parameters can be made again here with the data in Figure 4.

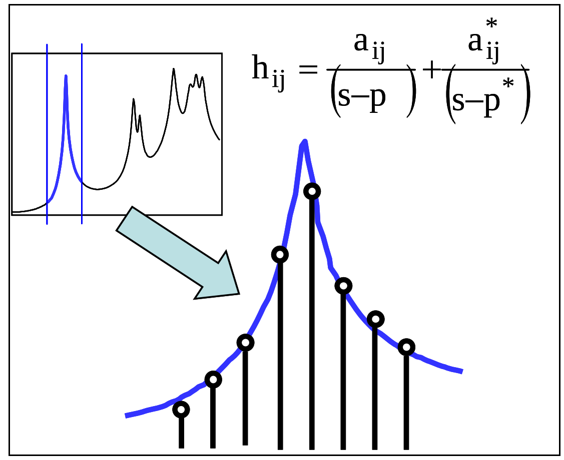


Figure 3 – Conceptual SDOF Curvefit

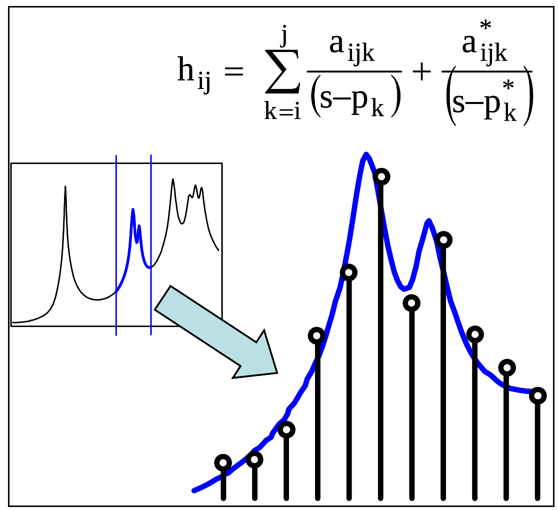


Figure 4 – Conceptual MDOF Curvefit

So if you accept the procedure that you always perform for the simple straight line fit, then you have to agree that the same procedure is applied for the modal parameter estimation process (but of course the data is complex and the line is slightly more complicated). Essentially in both cases, parameters are extracted, in a least squares fashion, that describe the function.

So there really isn't any black magic at all to the curvefitting process. It is really the same process that we all perform with simpler straight line regression analyses. Modal parameter estimation is just an extention of simpler curvefitting of data. If you have any other questions about modal analysis, just ask me.