MODAL SPACE - IN OUR OWN LITTLE WORLD by Pete Avitabile

Illustration by Mike Avitabile

If I run a shaker test with the input oblique to the global coordinate system, how do I decompose the force into the specific components in each direction? Wait – we need to discuss this before you take data.

Well it turns out that you don't really need to break the force up into components in the global coordinate system. There is an easier way to account for this. But first let's discuss a few basics before we get to how we are going to help fix the problem you described.

The first thing we need to understand is that even if you could break the input force up into two separate inputs in the global coordinate system, you wouldn't actually have two separate independent inputs; the two inputs are linearly related to the one independent input applied to the structure. So even if you could break it up into components there would be no advantage to doing that. But let's think about how we got ourselves into even thinking that we needed to decompose the force into separate components.

Let's start this discussion with a simple structure that has mode shapes that are very directional in nature as seen in Figure 1.

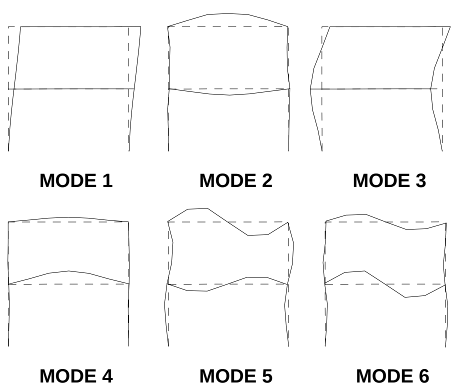

Figure 1 – Mode Shapes With Very Directional Character

Now just what do I mean by directional modes. That means that the response of the structure is primarily in one direction with very little or no response in the other directions for a given mode of the structure. Yet another mode of the structure may have response in a different direction than the first mode with little or no response in the other directions. We can see this in Mode 1 and Mode 3 in Figure 1; notice that the mode shapes are basically in the horizontal direction with very little motion in the vertical direction. But if we look at Mode 2, Mode 4, Mode 5, and Mode 6, we see that the main motion in the shape is in the vertical direction with little motion in the horizontal direction.

So if we wanted to pick a reference point on the structure for the modal test, then it isn't easy to do if I restrict myself to either the horizontal direction (X) or the vertical direction (Y). So maybe I would need to have some reference that is oblique to the global coordinate system that I selected for the set of measurements. For the purpose of this discussion, let's assume that I am only interested in the first four modes of the system. First let's write the equations assuming that I will have a reference in one modal test for the x-direction modes and then a reference in a second modal test for the y-direction modes. (Eventually, we will write the equations for an oblique reference to show how to select a reference that is suitable for all the modes in one modal test.)

The most important thing to discuss now is the importance of the drive point measurement and how it relates to the equations describing the residues and mode shapes. Let's recall the equation for the frequency response function

$$h_{ij}(j\omega) = \sum_{k=1}^{m} \frac{a_{ijk}}{(j\omega - p_k)} + \frac{a_{ijk}^*}{(j\omega - p_k^*)}$$

We need to remember that the residues are directly related to the mode shapes (and the q scaling factor) for a particular measured degree of freedom as

$$a_{ijk} = q_k u_{ik} u_{jk}$$

or for the whole set of measurements in matrix form as

$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

So if we picked a particular reference such as 7x and measured at 24 points in the x and y directions, then the set of data would be written for a particular mode as

$$\begin{Bmatrix} a_{1x7x} \\ a_{1y7x} \\ a_{2x7x} \\ a_{2y7x} \\ a_{3x7x} \\ \vdots \\ a_{7x7x} \\ \vdots \\ a_{24x7x} \\ a_{24y7x} \end{Bmatrix} = q\ u_{7x} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ \vdots \\ u_{7x} \\ \vdots \\ u_{24x} \\ u_{24y} \end{Bmatrix}$$

and then we would see that the drive point measurement at 7x would be the measurement needed to scale the residues to get scaled mode shapes using

$$a_{7x7x} = q\ u_{7x} u_{7x}$$

But we have to remember that from the reference in the x-direction (7x), only Mode 1 and Mode 3 can be easily measued because these modes are in the x-diection whereas Mode 2 and Mode 4 are modes in the y-direction and cannot be easily measured if the reference is in the x-direction.

In order to measure Mode 2 and Mode 4, a reference in the y-direction is necessary. Of course, a second test is needed to accomplish this. If a reference is selected at point 20Y for instance, then the equation would be written relative to that reference and the drive point at 20Y would be used to obtain a scaled mode shape as discussed above and is

$$\begin{Bmatrix} a_{1x20y} \\ a_{1y20y} \\ a_{2x20y} \\ a_{2y20y} \\ a_{3x20y} \\ \vdots \\ a_{20y20y} \\ \vdots \\ a_{24x20y} \\ a_{24y20y} \end{Bmatrix} = q\ u_{20y} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ \vdots \\ u_{20y} \\ \vdots \\ u_{24x} \\ u_{24y} \end{Bmatrix} \quad \text{and} \quad a_{20y20y} = q\ u_{20y} u_{20y}$$

But this requires that a modal test be run twice with two different references. Another approach would be to select an additional point on the structure at some oblique angle at an arbitrary point 99s for instance where a drive point measurement of force and acceleration is made. The input to the structure is shown for illustration in Figure 2. This reference can be any point on the structure at any oblique angle as long as that location is suitable to excite all the modes of interest.

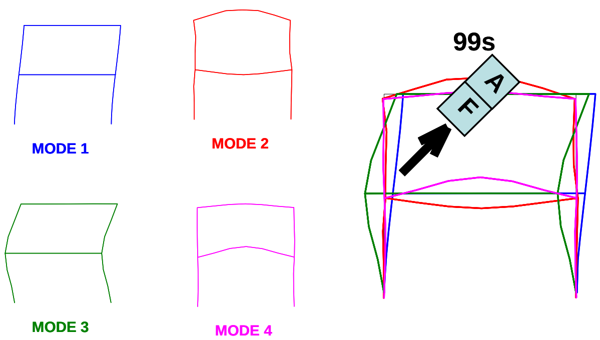

Figure 2 – Modal Test with Oblique Input Excitation

With this set of measurements, the set of equations relative to reference at point 99s and the drive point measurement would be

$$\begin{Bmatrix} a_{1x99s} \\ a_{1y99s} \\ a_{2x99s} \\ a_{2y99s} \\ a_{3x99s} \\ a_{3y99s} \\ a_{4x99s} \\ a_{4y99s} \\ \vdots \\ a_{99s99s} \end{Bmatrix} = q\ u_{99s} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \\ \vdots \\ u_{99s} \end{Bmatrix} \quad \text{and} \quad a_{99s99s} = q\ u_{99s} u_{99s}$$

So once the mode shapes are scaled using the drive point measurement, then there really is no need to include the reference point in the description of the mode shapes. This happens to be a very convenient way to obtain scaled mode shapes without ever needing to include the oblique drive point measurement in the actual geometry description of the mode shapes. But of course it is critical to remember that the oblique reference location must be a good location where all of the modes can be observed from that one reference location for this to work.

I hope this explanation helps you to understand that you can pick any angle for the reference - just as long as its not the node of a mode. And you can use this oblique reference location as a drive point for scaling the modes of the system. If you have any other questions about modal analysis, just ask me.