 by Pete Avitabile

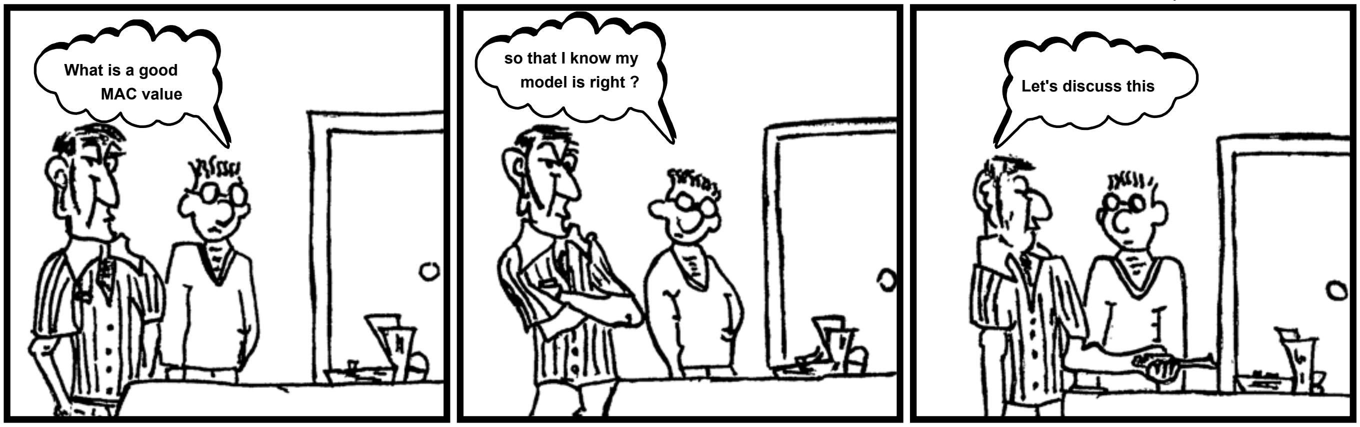

Illustration by Mike Avitabile

What is a good MAC value so I know my model is right ? Let's discuss this.

Now this is a question which needs a lot of discussion. Many people are often confused about MAC and the other correlation tools that are commonly used. There are a few issues to be discussed in order to clarify some misconceptions.

For purposes of discussion, let's assume that we have an analytical model and experimental data that has close to perfect vector correlation viewed from the MAC (Modal Assurance Criteria) and POC (Pseudo-Orthogonality Check); both approach the desired unity matrix. But, while the vectors correlated well, let's assume that the frequency correlation is not quite as good and assume that there is a 10% frequency variation for the first mode and only a 1% variation for the second mode. So what does this correlation mean then.

To help with this discussion, let's look at the response of a simple plate that has been discussed in several previous Modal Space articles to explain some simple concepts. Figure 1 shows the response of the plate due to a random excitation as the input excitation and corresponding output random response due to that input. Also shown is the frequency respresentation of that input-output phenomena.

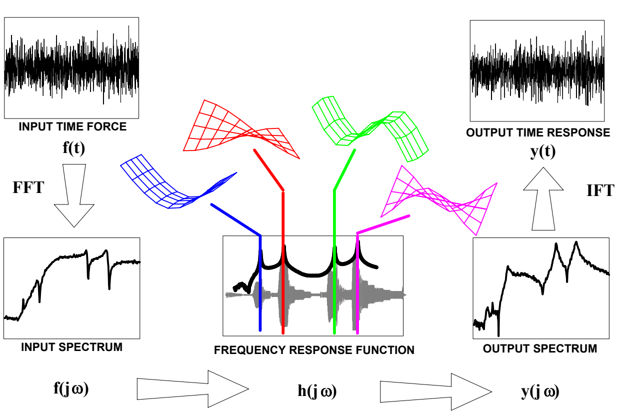

Figure 1 –Overall Response with Random Input Schematic

The FRF and impulse response is nothing more than a filter applied to the input excitation. The FRF is also shown with each of the corresponding mode shapes at each of the resonant frequencies. So we see that the frequency value as well as the mode shape is important for identifying the response of the system. While the shapes are correct, the frequency difference is also important.

If the frequency value is not correct then the response will vary depending on how the input spectrum varies. In this case the second mode frequency is very accurate and the input spectrum is fairly flat over the region of the second mode so the slight frequency variation only causes slight change in the response of the system.

However, for the first mode there is a 10% variation of the frequency. For this mode, there is a significant variation of the input frequency spectrum in this frequency range. So the variation of the frequency is more important for this mode than the second mode.

So it starts to become fairly obvious that the MAC is only an indicator of the vector correlation. But that only identifies if the vectors are correlated. It doesn't provide any information as to the suitability of the model to accurately predict the response of the system. But how does the vector affect the response. Well, the best way to understand the vector effect on the response is to look at the basic equation of motion.

The physical response of the system is

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\}$$

where [M], [C], [K] are the mass, damping and stiffness matrices respectively, along with the corresponding acceleration, velocity and displacement and the force applied to the system. This can be written in modal space as

$$\begin{bmatrix}\ddots & & \\ & \bar{M} & \\ & & \ddots\end{bmatrix}\{\ddot{p}\} + \begin{bmatrix}\ddots & & \\ & \bar{C} & \\ & & \ddots\end{bmatrix}\{\dot{p}\} + \begin{bmatrix}\ddots & & \\ & \bar{K} & \\ & & \ddots\end{bmatrix}\{p\} = [U]^T\{F\}$$

where the diagonal matrices are the modal mass, modal damping and modal stiffness along with the modal acceleration, modal velocity, and modal displacement. The right hand side of the equation has the modal force. Notice the the force is projected to modal space using the transpose of the modal vectors. So the mode shapes are important for the identification of the modal characteristics as well as the appropriation of the physical force to each of the modal oscillators.

If the mode shape varies then the distribution of load and response will vary. So we have to think about how we are going to use the model and more importantly we need to identify what types of loads will be applied and what response is critical to the overall performance of the system. With a random excitation that is broadband and fairly uniform in nature, some of these effects will generally be small.

Now to continue on with another example, Figure 2 shows a sinusoidal excitation with some harmonic components to the driving frequency. Notice that the driving frequency is NOT at one of the resonances of the system. But what if the model frequency was wrong? and the excitation was actually aligned to the first mode? Then there would be more response than predicted.

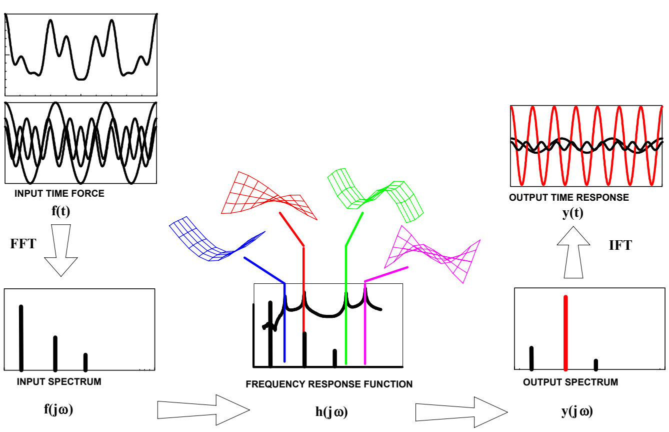

Figure 2 – Overall Response with Sine Input Schematic

And on the other hand, what would happen if the second mode frequency was wrong in the model? Notice that a harmonic of the driving frequency is aligned to the second mode of the system. The response would be predicted wrong.

So we have to start to think about what the MAC values mean in regards to the entire model and the response of that model. The MAC (and POC) helps us to identify how accurate the shape is. But we also need to think about the frequency correlation as well and the forcing function.

So when a correlation is performed, it is important to obtain the best correlation possible. But what does that really mean? There needs to be some assessment of the response of the model due to all the design loadings anticipated. Then someone needs to determine what variability can exist in the model and what effect that has on the computed response. It is then and only then that I can determine how much the frequencies and vectors can vary once someone has defined the acceptable variability in the model.

What we need to realize is that no model is ever perfect. Every model will have variation. And design loadings will have some variation also (if we consider real loading conditions). So before we can ever define levels of acceptable correlation, someone needs to define what is acceptable in terms of the overall system level response. If this isn't done then the levels specified for the frequency correlation and MAC/POC correlation are meaningless. If they are arbitrarily selected, then they may not really be good overall indicators as to how accurate the model prediction may be.

In certain applications there may a very strict requirement that the first and second modes MUST have very accurate frequency correlation as well as shape correlation if the loadings are very sensitive to the frequency of the signal. This is true in applications that involve rotating equipment where the specific operating speeds are critical to the overall response of the system. It may be more critical to have the frequency accurate in some instances and have less stringent requirements on the mode shape correlation. But in other applications where the inputs are uniform broadband excitations, then the frequency correlation may not be as critical and the shape correlation is more important.

This can not be simply identified in a fixed correlation specification. The simple fact is that the correlation and levels of acceptance need to be identified as a result of a detailed analysis of the system in question due to the specific loadings anticipated. Without this important evaluation, then the levels of correlation do not have any practical relevance.

Of course, it would be nice if the models developed satisfy some basic levels of correlation but that does not imply that the model will necessarily produce accurate results if these generic correlation levels are achieved.

I hope that this little discussion has shed some light on the correlation process and why the specific correlation values achieved must be used with an understanding that there needs to be some relationship to the response of the system. If you have any more questions on modal analysis, just ask me.